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Motion of dust in mean-motion resonances with planets

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Abstract. Effect of stellar electromagnetic radiation on motion of spherical dust particle in mean-motion orbital resonances with a planet is investigated. Planar circular restricted three-body problem with the Poynting-Robertson (P-R) effect yields monotonous secular evolution of eccentricity when the particle is trapped in the resonance. Elliptically restricted three-body problem with the P-R effect enables nonmonotonous secular evolution of eccentricity and the evolution of eccentricity is qualitatively consistent with the published results for the complicated case of interaction of electromagnetic radiation with nonspherical dust grain. Thus, it is sufficient to allow either nonzero eccentricity of the planet or nonsphericity of the grain and the orbital evolutions in the resonances are qualitatively equal for the two cases. This holds both for exterior and interior mean-motion orbital resonances. Evolutions of longitude of pericenter in the planar circular and elliptical restricted three-body problems are shown. Our numerical integrations suggest that any analytic expression for secular time derivative of the particle's longitude of pericenter does not exist, if a dependence on semi-major axis, eccentricity and longitude of pericenter is considered (the P-R effect and mean-motion resonance with the planet in circular orbit is taken into account).

Change of optical properties of the spherical grain with the heliocentric distance is also considered. The change of the optical properties: i) does not have any significant influence on secular evolution of eccentricity, ii) causes that the shift of pericenter is mainly in the same direction/orientation as the particle motion around the Sun. The statements hold both for circular and noncircular planetary orbits.

Key words. scattering, celestial mechanics, interplanetary medium

1. Introduction

Orbital motion of dust particles in the zones of mean-motion orbital resonances with a planet is intensively discussed since the paper by Jackson and Zook (1989) was published. A body is in resonance with a planet when the ratio of their mean motions (mean motion $n = 2\pi/T$, where T is orbital period) is the ratio of two small integers. Besides important gravitational attraction of the Sun and the planet, moving usually in circular orbit, also the effect of solar electromagnetic radiation on the particle is considered. Standardly, dust particle is considered to be spherically symmetric and, correspondingly, the effect of solar electromagnetic radiation is considered in the form of the Poynting-Robertson (P-R) effect: e. g., Jackson and Zook (1989), Šidlichovský and Nesvorný (1994), Beaugé and Ferraz-Mello (1994), Marzari and Vanzani (1994), Liou and Zook (1995), Liou *et al.* (1995), Liou and Zook (1997). Observations confirming the existence of dust ring around the Sun in resonant lock with the Earth are discussed in Brownlee (1994), Dermott *et al.* (1994), Reach *et al.* (1995). The paper by Liou and Zook (1997) presents secular orbital evolution of eccentricity when the particle is trapped in mean-motion orbital resonances (Liou and Zook 1997 – Eq. 26; see also Klačka and Kocifaj 2006a – Sec. 9, mainly Eq. 83). As a consequence, P-R effect and circular restricted three-body problem yield monotonous secular evolution of eccentricity of the particle trapped in a mean-motion orbital resonance with a planet. However, qualitatively new result was obtained by Klačka *et al.* (2005), when a general interaction of electromagnetic radiation with dust grain was taken into account. Really, arbitrarily shaped dust grain and its interaction with solar electromagnetic radiation does not yield, in general, monotonous secular evolution of eccentricity of the particle trapped in mean-motion resonances (Klačka *et al.* 2005a, 2005b; Klačka and Kocifaj 2006a, 2006b). Thus, an interesting question arises: Does exist any other generalization of the standardly used access (P-R effect + planet in circular orbit) which can produce results qualitatively consistent with nonspherical grains?

We concentrate on orbital evolution of spherical dust particles in mean-motion orbital resonances, in this paper. We take into account nonzero planetary eccentricity, also. Besides constant optical properties of the particle, also dependence of optical properties on heliocentric distance is considered, in accordance with Kocifaj *et al.* (2006). Influence of this effect is usually ignored in literature. In this paper we concentrate on orbital evolution of dust grains inside resonances: the initial conditions are given in the form that the particle starts its motion inside the resonances. Results of our study can be applied also for the case of orbital evolution of particles captured in the Earth resonant ring, which is well observed (Brownlee 1994, Dermott *et al.* 1994, Reach *et al.* 1995). We compare our results with the results obtained for nonspherical particles which were published elsewhere. It is shown that nonzero eccentricity of the planet can mimic behaviour of nonspherical dust grains for circular planetary orbit.

Attempts in dealing with restricted elliptic three-body problem can be found also in Gonczi *et al.* (1983). However, the authors ignore (see their Eq. 1) the inertial gravitational term and the nongravitational term represents only a partial component generated by the action of solar electromagnetic radiation (they would not receive, e. g., results on orbital evolution presented by Robertson 1937 and Wyatt and Whipple 1950). Thus, our equation of motion will be more general, and it will really correspond to physics. Moreover, we are motivated by the results obtained during the last ten years, which were not known at the time of Gonczi *et al.* (1983).

2. Mean-motion resonances with planets

According to the third Kepler's law we have

$$a^3 \, n^2 \; = \; G \, M_\odot \, (1 \; - \; \beta) \; ,$$
$$a_P^3 \, n_P^2 \; = \; G \, (M_\odot \; + \; m_P) \; , \tag{1}$$

where a, a_P are semi-major axes of a particle (characterized by optical parameter β) and a planet with mass m_P, n and n_P are mean motions characterizing revolutions around the Sun of the mass $M_\odot$. The first part of Eq. (1) uses the fact that central Keplerian acceleration is given by the sum of solar gravitational acceleration and radial component of radiation pressure acceleration. Eq. (1) yields

$$a = (1 \; - \; \beta)^{1/3} \; \left(\frac{n_P}{n}\right)^{2/3} \; \left(1 \; + \; \frac{m_P}{M_\odot}\right)^{-1/3} \; a_P \; . \tag{2}$$

A particle is in a mean-motion resonance with a planet when the ratio of their mean motions is the ratio of two small integers. If the dust particle is in the resonance with the planet, we can define the $q-$th order exterior resonance by the relation $n_P/n = (p+q)/p$, where p and q are integer numbers. Similarily, the $q-$th order interior resonance is defined by the relation $n_P/n = p/(p+q)$, where p and q are integer numbers. In terms of orbital periods: $T/T_P = (p+q)/p$ for exterior , $T/T_P = p/(p+q)$ for interior resonance. On the basis of these definitions and Eq. (2), we can immediately write

$$a = (1 \; - \; \beta)^{1/3} \; \left(\frac{p+q}{p}\right)^{2/3} \; \left(1 \; + \; \frac{m_P}{M_\odot}\right)^{-1/3} \; a_P \; , \tag{3}$$

for the semi-major axis of the dust particle in the $q-$th order exterior resonance with the planet of mass m_P. Similar relation can be obtained for the interior resonance.

3. Equation of motion for spherical dust particle

Let us consider a spherical dust grain under action of gravitational forces generated by the Sun and a planet moving around the Sun. Let the grain is under action of solar electromagnetic radiation, too. The considered electromagnetic radiation effect is known
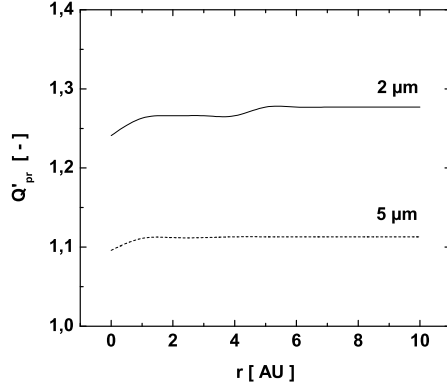
Fig. 1. Radiation pressure efficiency factor $\bar{Q}'_{pr}$ as a function of heliocentric distance for compact carbonaceous dust particles with radii $R = 2$ μm and $R = 5$ μm.

as the Poynting-Robertson effect. Equation of motion of the particle is considered in the form

$$\frac{d\,\boldsymbol{v}}{d\,t} = -\frac{GM_\odot\,(1-\beta)}{r^2}\,\boldsymbol{e}_R - \beta\,\frac{GM_\odot}{r^2}\left(\frac{\boldsymbol{v}\cdot\boldsymbol{e}_R}{c}\,\boldsymbol{e}_R + \frac{\boldsymbol{v}}{c}\right) -$$
$$- Gm_P\left\{\frac{\boldsymbol{r}-\boldsymbol{r}_P}{|\boldsymbol{r}-\boldsymbol{r}_P|^3} + \frac{\boldsymbol{r}_P}{|\boldsymbol{r}_P|^3}\right\}\,,$$
$$\beta = \frac{L_\odot \pi R^2}{4\pi\,GM_\odot m}\frac{\bar{Q}'_{pr}}{c} = 7.6\times 10^{-4}\,\bar{Q}'_{pr}\,\frac{\pi R^2\,[m^2]}{m\,[kg]}\,, \qquad (4)$$

where $\boldsymbol{r}$ and $\boldsymbol{r}_P$ are position vectors of the particle and the planet (mass m_P) with respect to the Sun (mass $M_\odot$), $r = |\boldsymbol{r}|$, $\boldsymbol{e}_R = \boldsymbol{r}/r$, $\boldsymbol{v} = d\boldsymbol{r}/dt$ is velocity of the particle, G is the universal gravitational constant, c is speed of light, m is mass of the particle ($m = 4\pi R^3\rho/3$ we consider homogenous particles with density ρ), R is radius of the particle, $\bar{Q}'_{pr}$ is the spectrally averaged efficiency factor for radiation pressure and $L_\odot$ is the solar luminosity. As for the Poynting-Robertson effect, we use equation of motion derived by Robertson (1937), see also Klačka (1992, 2000, 2004).

In this paper we consider Eq. (4) as the equation of motion in the elliptically restricted three-body problem together with the P-R effect. To obtain planetary position in elliptical orbit we solve Kepler equation. Initial conditions of the planet and the particle always correspond to prograde (counter-clockwise) motion, in our numerical integrations. Moreover, in some cases, we admit that optical properties of the particle may change with heliocentric distance due to the change of temperature. Change of optical properties is described by change of spectrally averaged efficiency factor for radiation pressure $\bar{Q}'_{pr}$. We have calculated the value of $\bar{Q}'_{pr}$ for particles with radii 2 μm and 5 μm in the range of heliocentric distances from 0 to 10 AU. Relation of $\bar{Q}'_{pr}$ on heliocentric distance is shown in Fig. 1. Method that we used for calculating spectrally averaged efficiency factor is taken from Kocifaj *et al.* (2006) and the results of Klačka *et al.* (2007).

4. Secular evolution of eccentricity in mean-motion resonances with planet in circular orbit

Secular evolution of semi-major axis is characterized by its constant value when the particle is in a resonance with a planet. What can be said about secular evolution of the eccentricity of the particle in such a situation? In this derivation we will suppose that the planet is moving in a circular orbit around the Sun. In this case, we have a special gravitational problem of three bodies and small nongravitational forces are also present. The gravitational problem is know as *the circular restricted three-body problem*. At the end of the 19-th century F. F. Tisserand found a quantity, which does not change during motion of the third body, whose mass is negligible in comparison to the masses of planet and the Sun (see, e. g. Brouwer and Clemence 1961). For our case, we can write the Tisserand's parameter in the form

$$C_T = \frac{1-\beta}{a} + 2 \sqrt{\frac{(1-\beta)\,a\,(1-e^2)}{a_P^3}} \, \cos I \; , \tag{5}$$

where e is eccentricity of the particle characterized by constant parameter β and I is the inclination of the particle's orbital plane with reference to the plane of the planet's orbit. We have to stress that Tisserand's quantity C_T does not change only for the special case of the circular restricted problem of three bodies. However, Eq. (5) enables to find secular change of the eccentricity of the particle captured in a resonance. For this purpose, we will consider $I = 0$ in Eq. (5). We can write

$$\frac{dC_T}{dt} = \frac{\partial C_T}{\partial a} \left(\frac{da}{dt}\right)_{total} + \frac{\partial C_T}{\partial e} \left(\frac{de}{dt}\right)_{total} \; , \tag{6}$$

for the total time derivative of the Tisserand quantity C_T defined by Eq. (5). However, according to Eq. (4), time derivatives of semi-major axis and eccentricity of the particle are caused by gravitational perturbations of the planet (these terms will be denoted by the subscript G) and nongravitational perturbations caused by Poynting-Robertson effect (these terms will be denoted by the subscript NG):

$$\left(\frac{da}{dt}\right)_{total} = \left(\frac{da}{dt}\right)_{G} + \left(\frac{da}{dt}\right)_{NG} \; ,$$
$$\left(\frac{de}{dt}\right)_{total} = \left(\frac{de}{dt}\right)_{G} + \left(\frac{de}{dt}\right)_{NG} \; . \tag{7}$$

On the basis of Eqs. (6)-(7) we can write

$$\frac{dC_T}{dt} = \frac{\partial C_T}{\partial a} \left\{ \left(\frac{da}{dt}\right)_{G} + \left(\frac{da}{dt}\right)_{NG} \right\} +$$
$$\frac{\partial C_T}{\partial e} \left\{ \left(\frac{de}{dt}\right)_{G} + \left(\frac{de}{dt}\right)_{NG} \right\} \; . \tag{8}$$

According to Tisserand, gravitational terms alone do not change the value of C_T:

$$\frac{\partial C_T}{\partial a} \left(\frac{da}{dt}\right)_{G} + \frac{\partial C_T}{\partial e} \left(\frac{de}{dt}\right)_{G} = 0 \; . \tag{9}$$

Putting Eq. (9) into Eq. (8):

$$\frac{dC_T}{dt} = \frac{\partial C_T}{\partial a} \left(\frac{da}{dt}\right)_{NG} + \frac{\partial C_T}{\partial e} \left(\frac{de}{dt}\right)_{NG} . \tag{10}$$

If we are interested in secular changes of orbital elements a and e, then the particle's stay in the resonance is characterized by the relation $\langle da/dt \rangle = 0$: for a function a of the property $a(T) = a(0)$ the relation $\langle da/dt \rangle = (1/T) \int_0^T (da/dt)dt = (a(T) - a(0))/T = 0$ holds. After averaging over period of the resonant oscillation of semi-major axis Eq. (6) reduces to

$$\left\langle \frac{dC_T}{dt} \right\rangle = \frac{\partial C_T}{\partial e} \left\langle \frac{de}{dt} \right\rangle . \tag{11}$$

Averaged Eq. (10) and Eq. (11) finally give for the total secular change of the eccentricity of the particle

$$\left\langle \frac{de}{dt} \right\rangle = \left\langle \frac{de}{dt} \right\rangle_{NG} + \frac{\partial C_T/\partial a}{\partial C_T/\partial e} \left\langle \frac{da}{dt} \right\rangle_{NG} . \tag{12}$$

Calculating partial derivatives of C_T defined by Eq. (5), and using relations for secular changes of the semi-major axis and eccentricity for the P-R effect (with assuming constant optical properties of particle) (Robertson 1937; Wyatt and Whipple 1950; Klačka 2004)

$$\left\langle \frac{da}{dt} \right\rangle_{NG} = -\beta \frac{GM_\odot}{c} \frac{2 + 3e^2}{a(1 - e^2)^{3/2}} , \tag{13}$$

$$\left\langle \frac{de}{dt} \right\rangle_{NG} = -\frac{5}{2} \beta \frac{GM_\odot}{c} \frac{e}{a^2(1 - e^2)^{1/2}} , \tag{14}$$

we get for the total secular change of the eccentricity (under the assumption that Eqs. (13)-(14) hold also for the period of resonant oscillation of the semi-major axis)

$$\left\langle \frac{de}{dt} \right\rangle = \beta \frac{GM_\odot}{c} \frac{\left(1 - e^2\right)^{1/2}}{a^2 \, e} \times X ,$$
$$X = 1 - \frac{\left(1 + 3e^2/2\right) \sqrt{1 - \beta}}{(a/a_P)^{3/2} \left(1 - e^2\right)^{3/2}} , \tag{15}$$

see Liou and Zook (1997), Klačka and Kocifaj (2006a, 2006b).

   Eq. (15) determines secular evolution of eccentricity of the spherical particle characterized by constant values β and $\bar{Q}'_{pr}$, if the particle is captured into a mean-motion resonance with planet moving in circular orbit. This equation enables to find detail evolution. If we take some special mean-motion resonance, we already know the value n_P/n and we can calculate a/a_P from Eq. (2). If the initial secular eccentricity $e < e_{\lim}$, where $e_{\lim}$ is given by Eq. (16) below, then the eccentricity of the particle is an increasing function of time, during the stay of the particle in the exterior mean-motion resonance. Eccentricity of the particle can only asymptotically approach to the limiting value $e_{\lim}$ given by the condition $X = 0$:

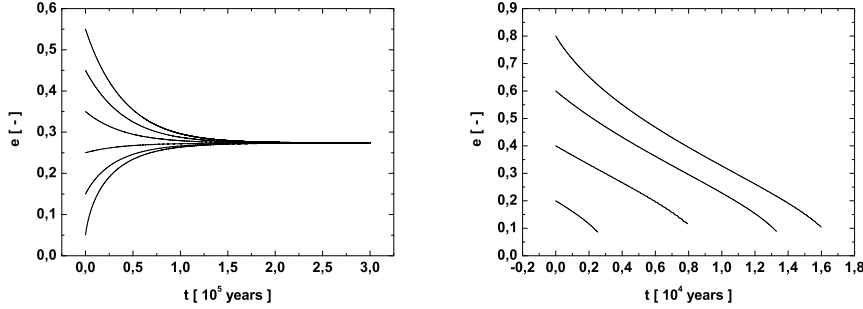$$\frac{p + q}{p} = \frac{1 + 3e_{\lim}^2/2}{(1 - e_{\lim}^2)^{3/2}} . \tag{16}$$

Fig. 2. Evolution of eccentricity of dust particle with $\beta = 0.01$ in resonances with a planet of mass equal to the Earth's mass, semi-major axis $a_P = 1$ AU and eccentricity $e_P = 0$. The left part is for 5/4 exterior mean-motion resonance. Various evolutions correspond to different initial values of eccentricity in the resonance. Initial eccentricities are: 0.55, 0.45, 0.35, 0.25, 0.15, 0.05. The right part of the figure is for 2/3 interior mean-motion resonance. Initial eccentricities are: 0.8, 0.6, 0.4, 0.2. The evolutions are numerically calculated from Eq. (4) and are consistent with Eq. (15).

If the initial eccentricity is greater than $e_{\lim}$, then $\langle de/dt \rangle$ is always negative and the eccentricity of the particle is a decreasing function of time, during the stay of the particle in the exterior mean-motion resonance. Eccentricity of the particle can only asymptotically approach to the limiting value $e_{\lim}$. Characteristic property of the value $e_{\lim}$ is that $e_{\lim}$ does not depend on β. The left part of Fig. 2 depicts evolutions of oscular eccentricity of dust particle with $\beta = 0.01$ in exterior resonance 5/4 with a planet of mass equal to the Earth mass, semi-major axis $a_P = 1$ AU and orbital eccentricity $e_P = 0$. The evolutions are calculated from numerical solution of Eq. (4). The first component of the right-hand side of Eq. (4) is used as a central acceleration. Asymptotical approach to the limiting value of eccentricity $e_{\lim} \approx 0.2736$ (given by Eq. 16) can be easily seen. The secular evolution of eccentricity is always a decreasing function of time for interior resonances defined by the relation $n/n_P = p/(p+q)$. Evolutions of oscular eccentricity of dust particle with $\beta = 0.01$ in the interior resonance 2/3 ($m_P = 1$ m_{Earth}, $e_P = 0$, $a_P = 1$ AU) is shown in the right part of Fig. 2. Evolutions of eccentricities for the given resonance and β are parallel – the evolutions are shifted along time axis since Eq. (15) yields the same value of $\langle de/dt \rangle$ for the same eccentricity e. If $\beta = 0$ (e. g., an asteroid) we have, from Eq. (15), $\langle de/dt \rangle = 0$.

The crucial question emerges: Are the above presented features typical for real dust particles and real physical situations?

Eq. (15) holds for circular planetary orbit and for the particle with constant optical properties in a mean-motion resonance with the planet. Evolution of eccentricity given by Eq. (15) will serve as a reference evolution. It will be used as a comparison with

the evolutions for the cases of non-circular planetary orbit and non-constant optical properties of the particle.

5. Secular evolution of longitude of pericenter in mean-motion resonances with a planet in circular orbit

Let us assume that a function of the type of Eq. (15) exists for secular evolution of the particle's longitude of pericenter if the particle is captured in a mean-motion orbital resonance with a planet in circular orbit in the planar case. Let us denote this function as S. The assumption is that for a given central star, the planet, the particle and the resonance, the function S depends on semi-major axis, eccentricity and longitude of pericenter of the particle:

$$\left\langle \frac{d\omega}{dt} \right\rangle = S(a, e, \omega) . \tag{17}$$

Fig. 3 depicts two evolutions of the orbital elements of the particles with $\beta = 0.01$ in the exterior resonance 5/4 with the planet of mass equal to the Earth's mass, semi-major axis $a_P = 1$ AU and eccentricity $e_P = 0$. The particle's initial values of the orbital elements for the first evolution, depicted by a solid line, are $a \approx 1.1565$ AU (given by Eq. 3), $e = 0.2$, $\omega = 90°$, and, for the second evolution, depicted by dashed line, $a \approx 1.1565$ AU, $e = 0.2$, $\omega = 54°$. At the time $t = 0$ the particles are at the perihelia of their orbits. The planet's initial position is at X-axis (axis, from which the longitude of the pericenter is measured), in the both integrations. Evolution of the secular eccentricity in Fig. 3 is consistent with the behavior expected from Eq. (15). We are interested in secular evolution of the longitude of pericenter. Evolutions of the longitude of pericenters intersect at the time $t \approx 614.3$ years. Secular values of the semi-major axes are practically identical, for the both orbits. The same holds for the eccentricities. However, the values of the $\langle d\omega/dt \rangle$ differ. This means that the function $S(a, e, \omega)$ does not exist for the given a, e and ω, since different values of $\langle d\omega/dt \rangle$ occur for the same a, e and ω.

Fig. 4 depicts evolution of the orbital elements of the particle with $\beta = 0.01$ in the exterior resonance 5/4 with the planet of mass equal to the Earth's mass, semi-major axis $a_P = 1$ AU and eccentricity $e_P = 0$. Particle's initial values are $a \approx 1.1565$ AU, $e = 0.2$, $\omega = 0°$ (X-axis) for both evolutions. The planetary initial position is at the X-axis in all integrations. Various particle's initial positions with respect to the planet yield different orbital evolutions. Various values of $\langle d\omega/dt \rangle$ exist during a short time interval at the beginning. This means, as in Fig. 3, that function S does not exist for the given initial values of a, e and ω. Although we have chosen only particular values of a, e and ω, we conjecture that $S(a, e, \omega)$ does not exist for any a, e and ω. It is impossible to uniquely define a function S for various initial conditions $\boldsymbol{r}$, $\boldsymbol{v}$, $\boldsymbol{r}_P$, $\boldsymbol{v}_P$ yielding a given values of a, e and ω. This does not mean that for the given values of a, e and ω there cannot exist two various sets of initial conditions leading to the same secular evolution of the
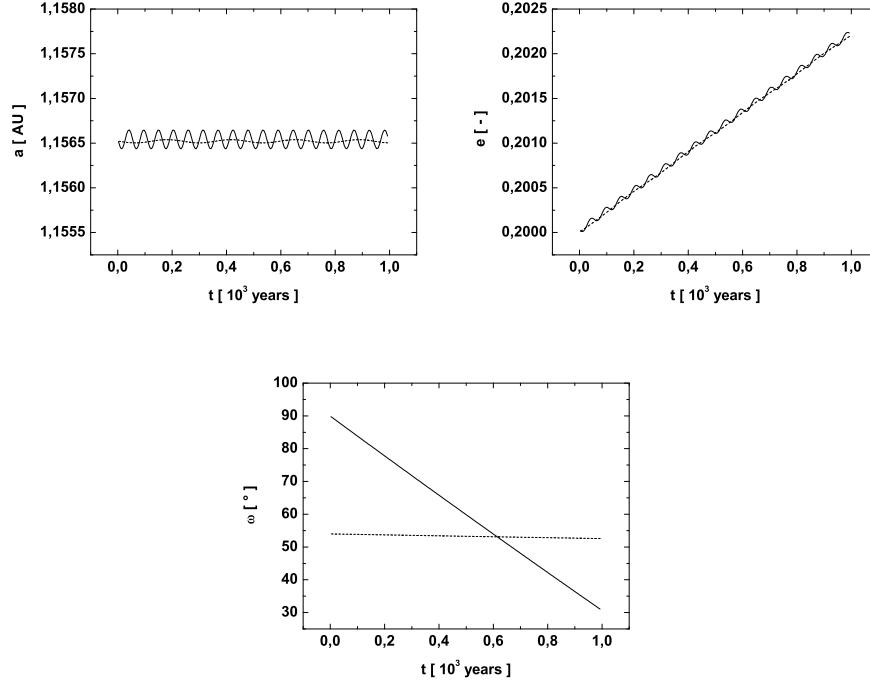
Fig. 3. Two orbital evolutions of dust particle with $\beta = 0.01$ captured in the exterior resonance 5/4 with a planet of the Earth's mass, semi-major axis $a_P = 1$ AU and eccentricity $e_P = 0$. Initial values of the first evolution are $a \approx 1.1565$ AU, $e = 0.2$, $\omega = 90°$ (solid line), for the second evolution $a \approx 1.1565$ AU, $e = 0.2$, $\omega = 54°$ (dashed line). Evolutions of the longitudes of pericenters intersect at time $t \approx 614.3$ years.

longitude of pericenter. Also in the case $\beta = 0$ we have found evolutions of the longitude of pericenter which have various $\langle d\omega/dt \rangle$ for the same a, e and ω. On the basis of this result we conjecture that the function S does not exist even in the case without the P-R effect when only perturbation from the planet is taken into account.

6. Secular evolution of eccentricity and shift of pericenter for noncircular planetary orbit

Fig. 5 shows evolution of semi-major axis, eccentricity and longitude of pericenter of spherical dust grain of constant optical properties with $\beta = 0.01$. The grain is captured in the exterior mean-motion resonance 5/4 with the planet of mass $m_P = 1\ m_{Earth}$ and semi-major axis $a_P = 1$ AU. Two values of planetary eccentricity are used: $e_P = 0$ and $e_P = 0.2$. Black solid line is used for the case $e_P = 0.2$, gray and dashed lines for $e_P = 0$. Initial eccentricity of the particle is 0.05. Evolution of the eccentricity for the circular planetary orbit is consistent with Eq. (15): evolution asymptotically approaches to the limiting value $e_{\lim} \approx 0.2736$ given by Eq. (16). Evolution of the particle's eccentricity is
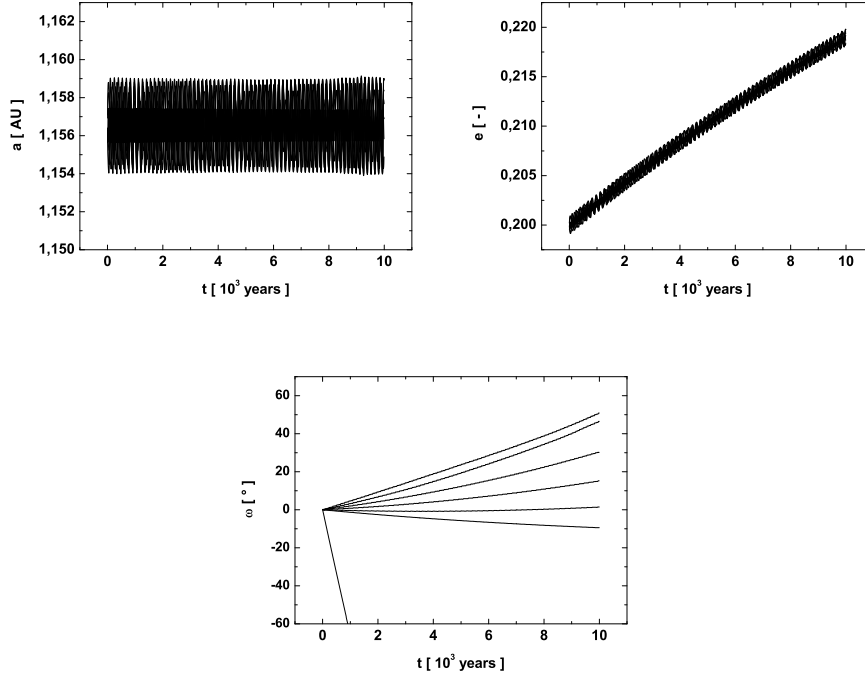
Fig. 4. Orbital evolutions of dust particle with $\beta = 0.01$ captured in the exterior resonance $5/4$ with a planet of the Earth's mass, semi-major axis $a_P = 1$ AU and eccentricity $e_P = 0$. Initial values of all evolutions are $a \approx 1.1565$ AU, $e = 0.2$, $\omega = 0°$. At the beginning, the particle is localized in the resonance and its initial position varies with respect to the planet.

a nonmonotonic function of time for the case $e_P = 0.2$. Fig. 5 shows that a limiting value of the grain eccentricity may not exist in the exterior resonance, if $e_P > 0$.

Fig. 6 depicts resonant evolution similar to Fig. 5. The main difference is that the black solid line holds for the eccentricity of the planet orbit $e_P = 0.0167$ (eccentricity of the Earth orbit), now. Moreover, initial conditions for the particle eccentricity are different: the initial value $e = 0.2$ for the case $e_P = 0.0167$ and $e = 0.1845$ for the case $e_P = 0$. The motivation was to obtain real oscillations of particle's eccentricity around the artificial, but analytically solvable (see Eq. 15), case given for $e_P = 0$. Period of the oscillations equals to the period of the shift of pericenter (period corresponding to the shift in $360°$). We have also found oscillatory evolution of the particle's eccentricity for an initial value $e > e_{\lim} \approx 0.2736$ for the $5/4$ resonance. The evolution of the particle's eccentricity oscillates around a curve which asymptotically decreases to the limiting value $e_{\lim} \approx 0.2736$ (see the left part of Fig. 2).

If the shift of pericenter is sufficiently fast, then oscillations in the evolution of secular eccentricity for exterior resonances exist. Period of this oscillations corresponds to the period of the shift of pericenter. This correspondence is caused by libration of conjunc-
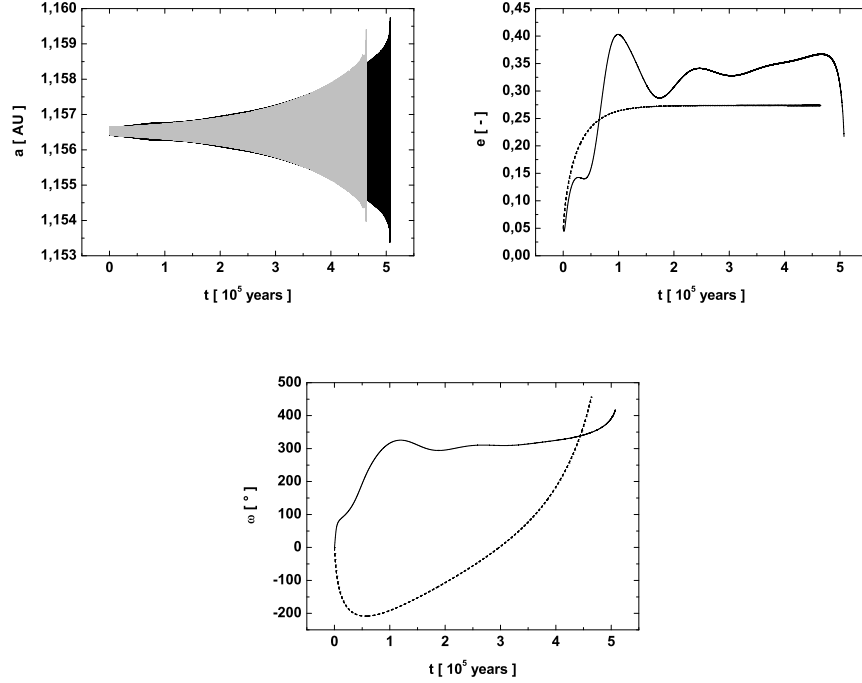
Fig. 5. Orbital evolution of dust particle with constant optical properties. Particle with $\beta = 0.01$ is captured in the exterior resonance $5/4$ with a planet of the Earth's mass, semi-major axis $a_P = 1$ AU and eccentricities $e_P = 0$ and $e_P = 0.2$. Initial eccentricity of the particle is 0.05. The case $e_P = 0.2$ corresponds to the black solid line. Gray solid line corresponds to the evolution of semi-major axis and dashed line to the evolution of eccentricity and longitude of pericenter. The case $e_P = 0.2$ does not lead to asymptotic behavior of eccentricity, in contrary to the case $e_P = 0$.

tions of the planet and the particle around aphelion of the particle's orbit. If the aphelion of the particle's orbit is shifted in $360°$, then the planetary orbit has the same position with respect to the aphelion of the particle orbit (we assume that secular eccentricity of the particle does not change significantly). This means that conjunctions of the particle and the planet take place in similar trajectories and this leads to similar change of the particle's eccentricity. If the shift of pericenter is slow, then oscillations in secular eccentricity do not exist. This situation is shown in Fig. 7.

Fig. 7 compares evolutions of orbital elements of dust particle with $\beta = 0.01$ captured in $5/4$ exterior resonance with the planet Earth and an "artificial Earth" moving in circular orbit. The shift of pericenter in Fig. 7 is much slower than the shift in Fig. 6. As a consequence of the slow shift of pericenter, the oscillations in evolution of eccentricity are not present in the first 2.5×10^5 years for the resonance with the Earth. Evolution of the pericenter during the first 2.5×10^5 is a nonmonotonous function of time: the initial
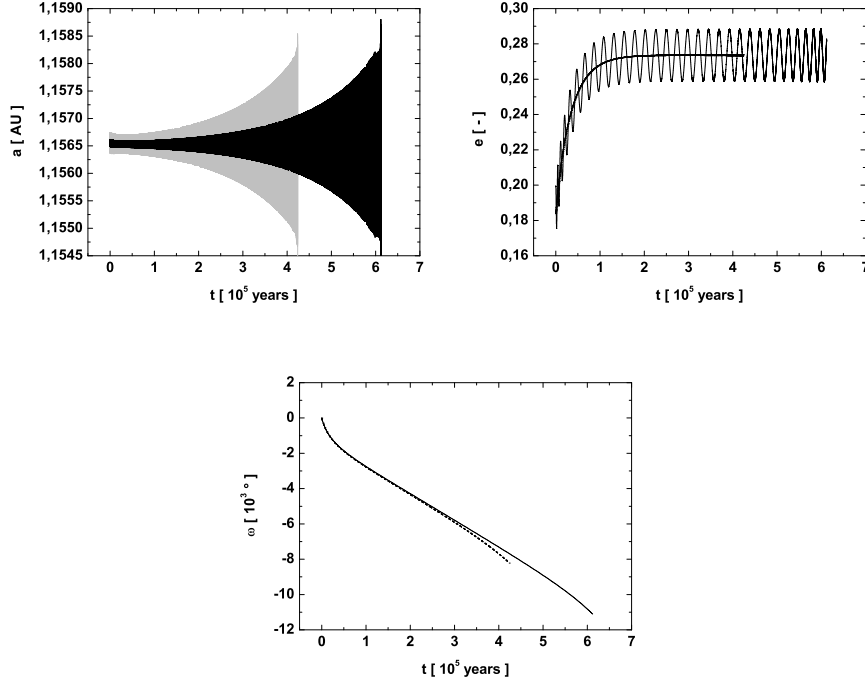
Fig. 6. Orbital evolution of dust particle with constant optical properties. Particle with $\beta = 0.01$ is captured in the exterior resonance $5/4$ with Earth (semi-major axis $a_P = 1$ AU and eccentricity $e_P = 0.0167$) – black solid line. Gray color line or dashed line represents evolution of particle orbital elements in the resonance with the planet in circular orbit. In the case of the resonance with the Earth, the secular evolution of eccentricity exhibits oscillations around $e(t)$ given by Eq. (15); period of oscillations corresponds to the period of the shift of pericenter.

decreasing function of time (for approximately 5×10^4 years) is followed by an increasing function.

Our simulations for exterior mean-motion orbital resonances of the first order show that maximal values of capture times – for a given β and a resonance $(j + 1)/j$ – for the case $e_P > 0$ are in several tens of percent greater than for the case $e_P = 0$. Also the mean capture time for $e_p > 0$ is greater than the mean capture time for $e_P = 0$. The same dependence holds for minimal capture times. This is different from the results for nonspherical dust grain (compare Klačka *et al.* 2005a, 2005b; Klačka and Kocifaj 2006a, 2006b). The greater aspect ratio and smaller volume of the nonspherical grain, the more important are nonradial radiation terms (terms with β_2 and β_3 in Eq. 41 in Klačka 2004) and the capture times in resonances are shorter. If the distance between the central star and the planet is greater, then the nonradial radiation terms are less important than the effect of the planet – see the term proportional to m_P in Eq. (4) – and the capture times are greater. These statements for the nonspherical particles are consistent with the
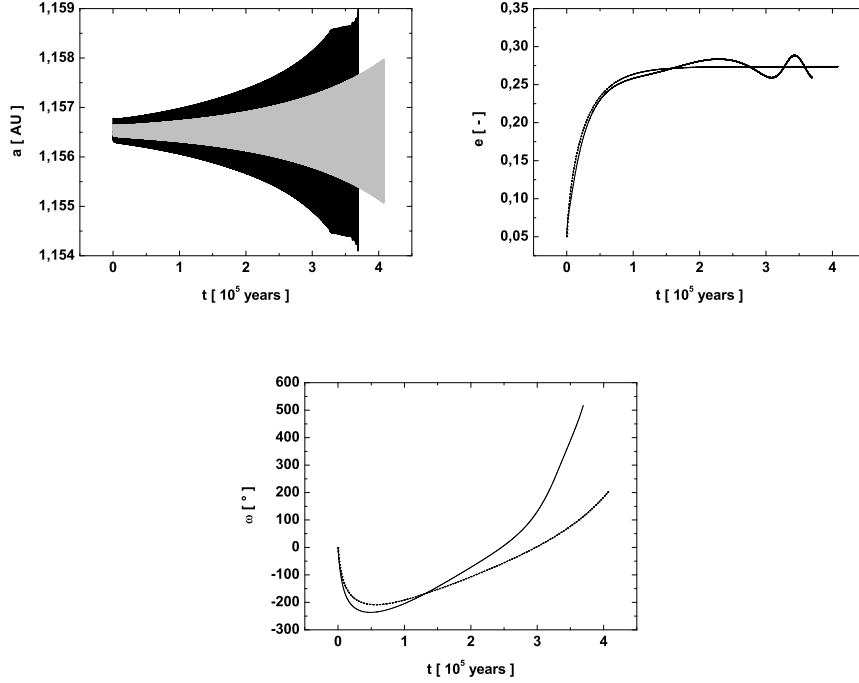
Fig. 7. Orbital evolution of dust particle with constant optical properties. Particle with $\beta = 0.01$ is captured in the exterior resonance 5/4 with Earth (semi-major axis $a_P = 1$ AU and eccentricity $e_P = 0.0167$) – black line. Gray color or dashed curve represents evolution of the orbital elements of the particle in the resonance with the planet in circular orbit. Initial conditions differ from those used in Fig. 6. The shift of pericenter is much slower than the shift in Fig. 6. Evolution of eccentricity exhibits only few oscillations at the end of the capture when the shift of pericenter is faster.

results of Klačka and Kocifaj (2002), Klačka *et al.* (2005a, 2005b), Klačka and Kocifaj (2006a, 2006b).

As for the interior mean-motion orbital resonances, we have found that noncircular planetary orbit can lead to secular increase of the dust grain eccentricity during the whole capture time; it seems that $e_P > e_P(critical) > 0$ is required, where $e_P(critical)$ depends on the type of the resonance, β and mass of the planet. This is different from the case $e_P = 0$, when only secular decrease of eccentricity exists (see Eq. 15). The situation is illustrated in Fig. 8, which holds for resonance 2/3, $\beta = 0.01$, $a_P = 1$ AU, $e_P = 0.2$ and planetary mass equal to the Earth mass. The obtained result is similar to the evolution of nonspherical particle with circular planetary orbit (compare Fig. 8 with Fig. 2 in Klačka *et al.* 2005b, or, with Fig. 3 in Klačka and Kocifaj 2006b). Fig. 8 shows also very complicated behavior of the shift of pericenter of the particle. Our simulations show that capture times of particles in the interior resonances are larger for $e_P \neq 0$ than for the case $e_P = 0$, if initial particles eccentricities are small. When eccentricity
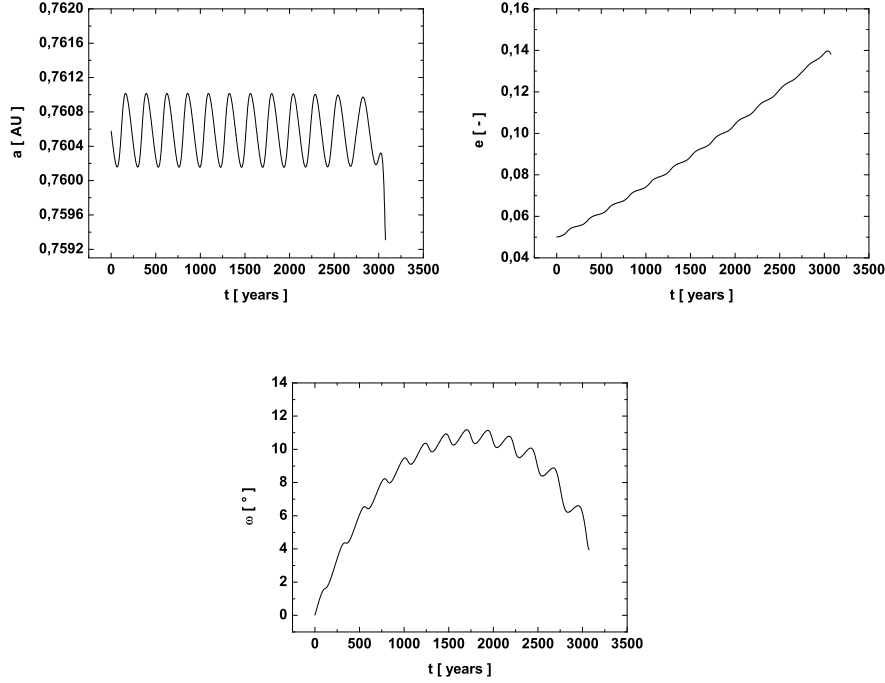
Fig. 8. Secular evolution of orbital elements of dust particle for the interior resonance 2/3 with a planet. Spherical particle of constant optical properties is characterized by $\beta = 0.01$. Mass of the planet equals the Earth mass, semi-major axis $a_P = 1$ AU and planetary eccentricity $e_P = 0.2$. Secular evolution of particle eccentricity is an increasing function of time during the capture in the resonance. Shift of pericenter is similar to the case $e_P = 0$.

of the particle in an interior resonance with the planet in circular orbit decreases to 0, the capture is ending because of lack of positive orbital energy given by the planet (Liou and Zook 1997). However, the eccentricity of the particle can be an increasing function of time, if the particle is captured in an interior resonance with the planet moving in elliptical orbit. The particle can approach to the planet and gain a sufficient amount of positive orbital energy to prevent a decrease of semi-major axis caused by the P-R effect. This is explanation of the longer capture times for $e_P \neq 0$, if initial particle eccentricities are small.

7. Influence of changing optical properties on evolution of orbital elements in mean-motion resonances

We are interested in orbital evolution of a particle with changing optical properties. As a central acceleration we will use $-GM_\odot(1 - \beta_0)\boldsymbol{e}_R/r^2$, where β_0 is the value of β at the time $t = 0$.

We will calculate time derivative of semi-major axis and eccentricity from perturbation equations of celestial mechanics for Eq. (4) without perturbation of a planet. We get

$$\frac{da_{\beta_0}}{dt} = \frac{2a_{\beta_0}e_{\beta_0}}{1-e_{\beta_0}^2} \sqrt{\frac{p_{\beta_0}}{GM_\odot(1-\beta_0)}} \, (\beta-\beta_0)\frac{GM_\odot}{r^2}\sin f_{\beta_0} - $$
$$- \frac{2a_{\beta_0}}{1-e_{\beta_0}^2} \frac{\beta GM_\odot}{cr^2} \left[2(e_{\beta_0}\sin f_{\beta_0})^2 + (1+e_{\beta_0}\cos f_{\beta_0})^2\right] \, , \tag{18}$$

$$\frac{de_{\beta_0}}{dt} = \sqrt{\frac{p_{\beta_0}}{GM_\odot(1-\beta_0)}} \, (\beta-\beta_0)\frac{GM_\odot}{r^2}\sin f_{\beta_0} - $$
$$- \frac{\beta GM_\odot}{cr^2} \left[e_{\beta_0}\sin^2 f_{\beta_0} + 2(e_{\beta_0}+\cos f_{\beta_0})\right] \, , \tag{19}$$

where a_{β_0}, e_{β_0} and f_{β_0} are oscular semi-major axis, eccentricity and true anomaly, $p_{\beta_0} = a_{\beta_0}(1-e_{\beta_0}^2)$ is parameter for elliptical orbit and $r = p_{\beta_0}/(1+e_{\beta_0}\cos f_{\beta_0})$. Secular evolution of semi-major axis and eccentricity can be obtained by averaging over one orbital period of the type

$$\langle g \rangle = \frac{1}{2\pi a_{\beta_0}^2 \sqrt{1-e_{\beta_0}^2}} \int_0^{2\pi} r^2 g(f_{\beta_0})df_{\beta_0} \, , \tag{20}$$

where g is any quantity. β is even function of true anomaly f_{β_0}. Thus, the average value of the first terms in Eqs. (18)-(19) is zero. We can write

$$\left\langle \frac{da_{\beta_0}}{dt} \right\rangle = - \frac{1}{\pi a_{\beta_0}(1-e_{\beta_0}^2)^{3/2}} \frac{GM_\odot}{c} \times$$
$$\times \int_0^{2\pi} \beta \left[2(e_{\beta_0}\sin f_{\beta_0})^2 + (1+e_{\beta_0}\cos f_{\beta_0})^2\right]df_{\beta_0} \, , \tag{21}$$

$$\left\langle \frac{de_{\beta_0}}{dt} \right\rangle = - \frac{1}{2\pi a_{\beta_0}^2 \sqrt{1-e_{\beta_0}^2}} \frac{GM_\odot}{c} \times$$
$$\times \int_0^{2\pi} \beta \left[e_{\beta_0}\sin^2 f_{\beta_0} + 2(e_{\beta_0}+\cos f_{\beta_0})\right]df_{\beta_0} \, . \tag{22}$$

β is approximately constant in Eqs. (21)-(22), see Fig. 1 (for more details about this approximation see Klačka *et al.* 2007). We get for secular evolution

$$\left\langle \frac{da_{\beta_0}}{dt} \right\rangle \approx - \beta_0 \frac{GM_\odot}{c} \frac{2+3e_{\beta_0}^2}{a_{\beta_0}(1-e_{\beta_0}^2)^{3/2}} \, , \tag{23}$$

$$\left\langle \frac{de_{\beta_0}}{dt} \right\rangle \approx - \frac{5}{2} \beta_0 \frac{GM_\odot}{c} \frac{e_{\beta_0}}{a_{\beta_0}^2(1-e_{\beta_0}^2)^{1/2}} \, . \tag{24}$$

Eqs. (23)-(24) are identical to Eqs. (13)-(14). Inserting Eqs. (23)-(24) into Eq. (12) (with the assumption that Eqs. (23)-(24) hold also for period of resonant oscillation of semi-major axis), we obtain equation which is identical to Eq. (15). This means that the change of optical properties does not significantly influence the evolution of secular eccentricity in mean-motion orbital resonances.

Now, we will derive an expression for secular evolution of the longitude of pericenter for Eq. (4) without action of a planet. Perturbation equations of celestial mechanics yield

$$
\frac{d\omega_{\beta_0}}{dt} = - \frac{1}{e_{\beta_0}} \sqrt{\frac{p_{\beta_0}}{GM_\odot(1 - \beta_0)}} (\beta - \beta_0)\frac{GM_\odot}{r^2} \cos f_{\beta_0} +
$$
$$
+ \frac{1}{e_{\beta_0}}\beta\frac{GM_\odot}{cr^2} \sin f_{\beta_0}(e_{\beta_0} \cos f_{\beta_0} - 2) . \tag{25}
$$

After averaging, using also Eq. (20), we have

$$
\left\langle \frac{d\omega_{\beta_0}}{dt} \right\rangle = - \frac{GM_\odot}{2\pi a_{\beta_0}^2 e_{\beta_0} \sqrt{1 - e_{\beta_0}^2}} \sqrt{\frac{p_{\beta_0}}{GM_\odot(1 - \beta_0)}} \times
$$
$$
\times \int_0^{2\pi} \beta \cos f_{\beta_0} df_{\beta_0} . \tag{26}
$$

Since β is an even function of true anomaly f_{β_0}, the second term in the right-hand side of Eq. (25) equals zero, after averaging. Also the average value of the term proportional to β_0 is zero. The shift of pericenter caused by the change of optical properties is in the same direction as the particle orbits the Sun, since β is an increasing function of heliocentric distance (see Fig. 1).

Fig. 9 depicts secular evolution of orbital elements of the particle with changing optical properties. Solid black line is used for spherical particle with radius $R = 5$ μm and density $\rho = 2$ g/cm^3 captured in 4/3 exterior resonance with the Earth. Gray line or dashed line is used for evolutions of the particle captured in the resonance with the planet of mass equal to the Earth mass, semi-major axis $a_P = 1$ AU and eccentricity $e_P = 0$. Evolution of eccentricity in Fig. 9 is similar to evolution of eccentricity in Fig. 6. Evolution of the eccentricity for the case $e_P = 0$ asymptotically approaches to the limiting value $e_{\text{lim}} \approx 0.3108$ given by Eq. (16). Evolution of eccentricity for the resonance with the Earth exhibits oscillations around the evolution for the resonance with circular planetary orbit. Period of oscillations corresponds to the period of the shift of pericenter. Evolution of the longitude of pericenter exhibits an increase in time. This is the difference from the behavior presented in Fig. 6. Behavior of the longitude of pericenter in Fig. 9 is caused, probably, by the influence of the changing optical properties, since the motion of the particle is prograde.

Fig. 10 compares evolution of a particle of constant optical properties with the evolution of another particle of changing optical properties. Both particles are characterized by the same initial conditions, as for the orbital elements and position with respect to the planet. Mass of the planet is equal to the mass of the Earth, semi-major axis $a_P = 1$ AU and eccentricity $e_P = 0$. The particles are captured in 3/2 exterior resonance with the planet. Fig. 10 presents that the shift of pericenter is in positive direction/orientation, for the case of changing optical properties of the particle. This is in coincidence with Eq. (26), since the motion is prograde. The shift of pericenter is in negative direction for particle with constant optical properties.
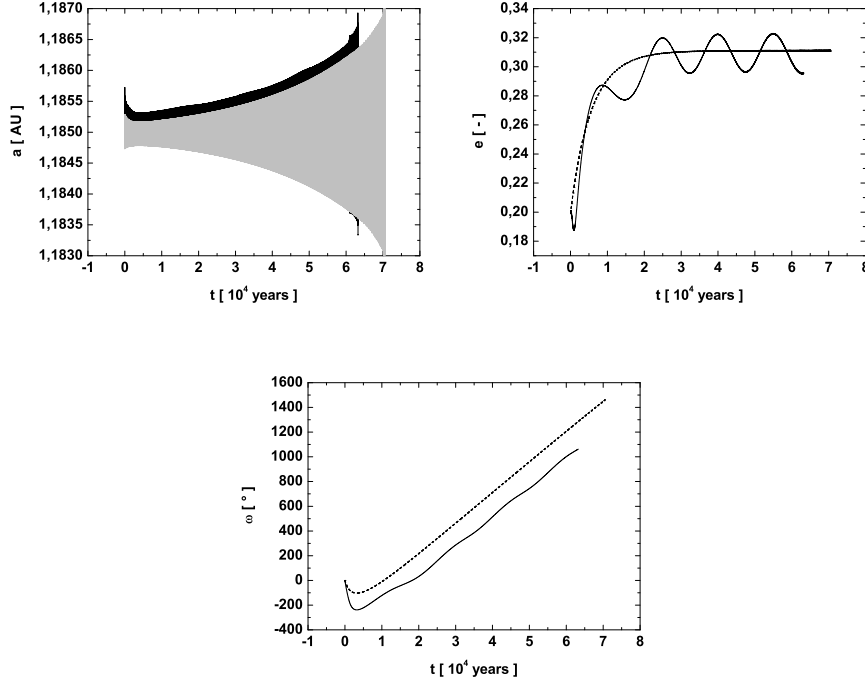
Fig. 9. Orbital evolution of dust particle with optical properties varying with heliocentric distance; radius of the particle $R = 5$ μm, density $\rho = 2$ g/cm^3. The particle is captured in the exterior resonance 4/3 with Earth (semi-major axis $a_P = 1$ AU and eccentricity $e_P = 0.0167$) – solid black line. Gray color or dashed curve represents orbital evolution of the particle in the resonance with the planet in circular orbit ($e_P = 0$). In the case of the resonance with the Earth, the evolution of eccentricity shows oscillations around $e(t)$ given by Eq. (15), with period of the shift of pericenter. Greater part of the shift of pericenter exhibits an increase in time and this differs from the behavior presented in Fig. 6.

It is also possible to find the shift of pericenter in negative direction for particle with radius $R = 2$ μm and density $\rho = 2$ g/cm^3. This case is depicted in Fig. 11. However, the shift of pericenter in negative direction is rare, for this type particle: the situation happens for initial conditions shown in Fig. 12.

As for the shift of pericenter, we may use perturbation equations of celestial mechanics. Central acceleration will be $-GM_\odot(1 - \beta_0)\boldsymbol{e}_R/r^2$, where β_0 is the value of β at time $t = 0$. Subtracting the central acceleration from the right-hand side of Eq. (4) yields the perturbation acceleration. Again, the planar circular restricted three-body problem and the P-R effect are considered. We get

$$\frac{d\omega_{\beta_0}}{dt} = -\frac{1}{e_{\beta_0}} \sqrt{\frac{p_{\beta_0}}{GM_\odot(1-\beta_0)}}\ (\beta - \beta_0)\frac{GM_\odot}{r^2}\ \cos f_{\beta_0} +$$

$$+ \beta\ \frac{GM_\odot}{r^2}\ \frac{1}{c}\ \frac{1}{e_{\beta_0}}\ (e_{\beta_0} \cos f_{\beta_0}\ -\ 2)\ \sin f_{\beta_0}\ -$$
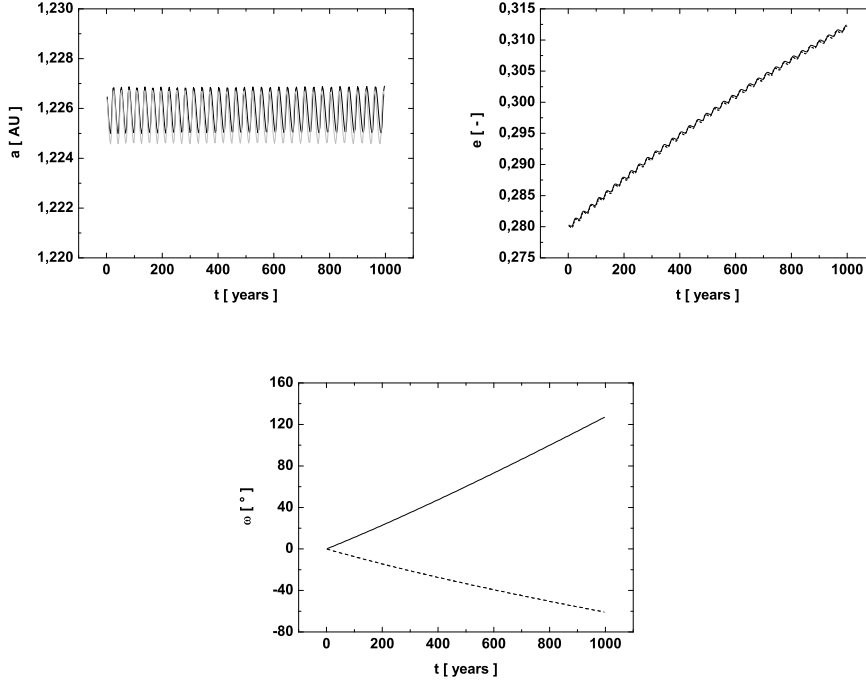
Fig. 10. Black line depict orbital evolution of dust particle with optical properties dependent on heliocentric distance; radius of the particle $R = 2$ μm, density $\rho = 2$ g/cm^3. Particle is captured in the exterior resonance $3/2$ with a planet of Earth mass, semi-major axis $a_P = 1$ AU and eccentricity $e_P = 0$. The shift of pericenter is in positive direction. Gray color or dashed curve represents orbital evolution of the particle in the resonance with the planet (identical initial conditions), but optical properties of the particle are constant. The shift of pericenter is in negative direction for particle with constant optical properties.

$$
\begin{aligned}
&\quad -\frac{Gm_P}{e_{\beta_0}}\sqrt{\frac{a_{\beta_0}(1-e_{\beta_0}^2)}{GM_\odot(1-\beta_0)}} \times Y \ , \\
Y &= -\frac{r}{|\boldsymbol{r}-\boldsymbol{r}_P|^3}\cos f_{\beta_0} + \left(\frac{r_P}{|\boldsymbol{r}-\boldsymbol{r}_P|^3}-\frac{1}{r_P^2}\right)\times Z \ , \\
Z &= \frac{\sin f_{\beta_0}}{1+e_{\beta_0}\cos f_{\beta_0}}\sin(\Theta_{\beta_0}-\Theta_P)+\cos(\Theta_P-\omega_{\beta_0}) \ ,
\end{aligned}
\tag{27}
$$

where $\Theta_{\beta_0} = \omega_{\beta_0} + f_{\beta_0}$ is the position angle of the particle on the orbit (measured from an X-axis) and Θ_P is the position angle of the planet on the orbit (measured from the X-axis).

Initial conditions depicted in Fig. 12 are: $\beta = \beta_0$, $f_{\beta_0} = 0°$, $\Theta_{\beta_0} = 0°$, $\omega_{\beta_0} = 0°$, $\Theta_P = 0°$, and r is a little less than r_P. Inserting the initial conditions into Eq. (24), one obtains $d\omega_{\beta_0}/dt < 0$ °/year. If r is a little greater than r_P, then $d\omega_{\beta_0}/dt > 0$ °/year. When the particle, is approximately, in the position depicted in Fig. 12, then the first term in Eq. (27) is negative – β is an increasing function of heliocentric distance and
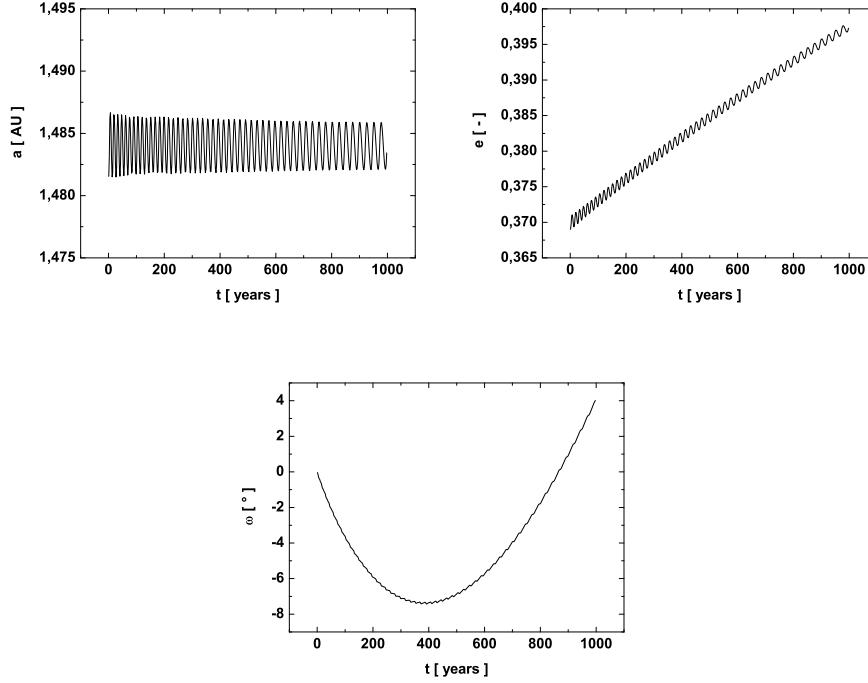
Fig. 11. Evolution of orbital elements of dust particle which optical properties depend on heliocentric distance. Radius of the particle $R = 2$ μm, density $\rho = 2$ g/cm^3. The particle is captured in the exterior resonance 2/1 with a planet of mass equal to the Earth mass, semi-major axis $a_P = 1$ AU and eccentricity $e_P = 0$. Shift of pericenter is in negative direction for $t \lesssim 400$ years. This situation happens for initial conditions shown in Fig. 12.

β_0 is the value of β at perihelion of the particle's orbit ($t = 0$). The sign of the second term depends on the sign of $\sin f_{\beta_0}$ and the third term is negative when the particle's position corresponds to that depicted in Fig. 12. The third term is dominant and the shift of pericenter is negative, since $\beta \approx \beta_0$, $f_{\beta_0} \approx 0°$ and the particle is near the planet. The oscular evolution of the longitude of pericenter is negative when the particle is in the interior part of the planetary orbit and the conjunction is near the particle's perihelion. This effect is relevant also for the secular evolution of the longitude of pericenter (shift of pericenter): when the particle moves outside the planetary orbit, then oscular evolution of the longitude of pericenter may be positive, but averaging over the orbital period yields negative sign. This may be explanation of Fig. 11 which depicts a decreasing secular evolution of the longitude of pericenter.

If the motion of dust particle is retrograde and the motion of the planet is prograde, then the shift of the particle pericenter should be, mainly, in negative direction, for the changing optical properties of the particle (Klačka *et al.* 2007). However, this depends on the detail behavior of the change of optical properties.
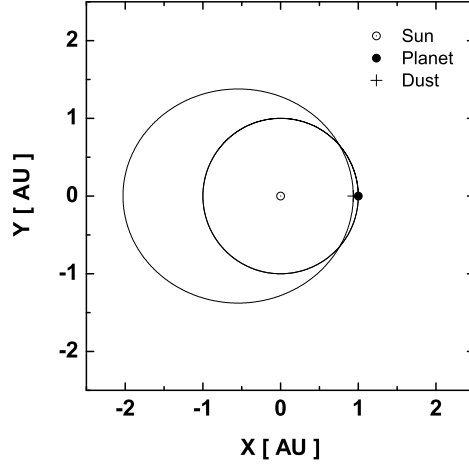
Fig. 12. Initial conditions for orbital evolution shown in Fig. 11 in the orbital plane XY. Ellipse represents trajectory of dust particle during the first two years. Sun is at the origin. Circle represents trajectory of the planet. Black cross represents the initial conditions of the particle. Black dot on the right represents initial position of the planet. The particle is initially inside the planetary orbit, but it is captured in the exterior resonance.

8. Conclusions

The contribution deals with the effect of solar electromagnetic radiation on dynamics of cosmic dust particles in mean-motion orbital resonances with a planet of mass equal to the mass of the Earth. We discuss not only the planar circular restricted three-body problem and the Poynting-Robertson effect with constant optical properties of the spherical particle. We admit also nonzero eccentricity of the planet and the change of particle optical properties with heliocentric distance. The paper concentrates on important properties of motion of dust grain in the zone of mean-motion resonances. We are interested mainly in pericenter motion and evolution of eccentricity in the resonances.

Our numerical integrations suggest that any analytic expression for secular time derivative of the particle's longitude of pericenter does not exist, if a dependence only on semi-major axis, eccentricity and longitude of pericenter is considered (P-R effect and mean-motion resonance with planet in circular orbit are taken into account).

If planetary eccentricity is close to zero and the shift of pericenter is sufficiently fast, then oscillations of dust grain secular eccentricity exist. The oscillations occur around the curve corresponding to secular evolution of the grain eccentricity in the planar circular restricted three-body problem with the P-R effect. This holds for exterior mean-motion orbital resonances. Nothing like this was found in the case of interior resonances. However, interior resonances can exhibit systematic increase of secular eccentricity of the grain

during the capture in the resonances. This is true when eccentricity of the planet is larger than some critical value depending on the type of the resonance, mass of the planet and optical parameter β. The case $e_P = 0$ yields only secular decrease of eccentricity, for interior resonances.

Our numerical simulations show that noncircularity of the planetary orbit stabilizes motion of spherical dust grain in the mean-motion orbital resonances. Maximal capture time (and also mean capture time for many captures in numerical simulations) of the grain for a given resonance and particle is greater than it is for the case of circular planetary orbit. This holds both for exterior and interior resonances. Nonsphericity of the grain destabilizes motion in the resonances: the greater aspect ratio and smaller volume of the grain, the shorter capture time.

If a change of optical properties of the spherical grain with heliocentric distance is also considered, then the shift of pericenter is dominated in positive direction/orientation for prograde motion of the particle; this holds both for circular and noncircular planetary orbits and exterior mean-motion orbital resonances (see, e. g., Fig. 10). If planetary orbit is characterized by large eccentricity, then secular evolution of dust grain eccentricity may exhibit complicated behavior.

Spherical dust grain in the planar circular restricted three-body problem with the Poynting-Robertson effect is characterized by a monotonic secular evolution of the grain eccentricity (this is true both for constant and variable optical properties of the grain), in the exterior mean-motion orbital resonances. Such kind of behavior does not exist if at least one of the above mentioned assumptions – sphericity of the grain or circular orbit of the planet – is cancelled. As for the interior resonances, the circular planetary orbit yields secular decrease of the eccentricity, while nonsphericity of the grain or noncircularity of the planetary orbit may yield secular increase of the eccentricity, also. There is some kind of unification of qualitative kinematical behavior of interplanetary dust grains under the action of more real physical forces: there is not great difference between evolution of orbital elements of spherical and nonspherical dust grains if more general physical forces are taken into account. Our results show that gravitational effect can mimic nongravitational effect. However, the following question is still unanswered: Can spherical grain be captured into a resonance when the secular evolution of particle semi-major axis is an increasing function of time?

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